

The New Micro Approach to Exchange Rates: Problem Set Solution

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(1) Wealth and CARA. Show that in a one-period CARA-Normal model with one risky asset that the risky-asset demand is independent of the level of wealth. Given that end-of-period wealth is a function of both risky-asset demand and the final payoff on the risky asset, and given that the former may be a function of a random signal, why is it OK to drop out the term that you do in your derivation above?

See the textbook (“The Microstructure Approach to Exchange Rates”) for the proof (pages 108-109). It is OK to drop that term because, from the perspective of the investor, his own risky asset demand is no longer a random variable since he has already observed the random signal when determining his demand.

(2) Risky-Asset Demands. Suppose you design a two-period CARA-Normal model and you reduce the agents’ optimization problem to the following expression:

$$\underset{D_{i1}}{\text{Max}} - \int_{-\infty}^{\infty} \exp \left[-\theta D_{i1} (I D_{i1} + I C_i + I X) - \frac{1}{2} (I D_{i1} + I C_i + I X)^2 \Sigma_2^{-1} - \frac{1}{2} \Sigma_X^{-1} X^2 \right] dX$$

where θ is the coefficient of absolute risk aversion, λ is a positive constant, D_{i1} is the risky-asset demand of agent i in period 1, C_i is a payoff signal received by agent i before trading in period 1, and X is the marketwide order flow observed at the end of period 1 (the only random variable in the expression). The terms Σ_2^{-1} and Σ_X^{-1} denote the inverse (conditional) variance of the period-two payoff and period-one order flow, respectively (both known). Solve for the risky-asset demand.

The solution to this problem follows the “Derivation of Result 2” in the textbook (pages 109-111) exactly. Once you have factored the integral and taken the first-order condition, you should find that the risky asset demand is proportional to C_i .

(3) Conditional Distributions. Suppose a signal x is normally distributed about a future price p . Specifically, let $x = p + z$, $p \sim N(0, m^{-1})$, and $z \sim N(0, n^{-1})$. Using the relation $\text{Var}[p|x] = E[p - E[p|x]]^2$, show that $\text{Var}^{-1}[p|x] = m + n$. In words, show that with normally distributed random variables, the variance of the posterior distribution is the inverse of the sum of the precisions.

The proof of this also appears in the course textbook (pages 111-112). This is a very useful result for microstructure modeling since most models focus on the following problem: What is $E[p|\text{order flow and other stuff}]$, where the conditioning variables correspond to the x ’s in the problem set question. The variances of those expectations are important to the modeling, so a convenient form for them is valuable.

(4) Information and Endowments. In chapter 4 of the MAER text (section 4.1, page 72), the uninformed trader does not take account of the effect of his risky-asset endowment on price. Should he? Is this treatment consistent with Grossman and Stiglitz (1980)? How would you defend this in a seminar?

To start with the second question, yes, it is consistent with Grossman and Stiglitz (1980). It is not, however, strictly kosher in the sense that these risky-asset endowments do affect price in equilibrium, and a rational

(non-infinitesimal) trader should take account of this, whether informed or uninformed. The assumption of non-strategic behavior rules out the taking of this effect into account, but it is precisely that assumption that is the stretch here. In any event, as footnote 14 points out, it is easy to finesse this as a technical matter by assuming that the informed and uninformed traders are represented by (separate) continuums of traders. In this setting, no single investor has a measurable effect on price.

(5) Kyle Models. Consider the single-period version of the Kyle (1985) model in section 4.2 of MAER. Let V denote the risky-asset payoff, and let D^I and D^U denote the informed- and uninformed-trader demands, respectively. Show that an informed trader who faces a marketmaker with the pricing strategy $P = \lambda(D^I + D^U)$ would trade according to the rule $D^I = \beta V$, where the parameters λ and β take the form $\lambda = 1/2(\Sigma_V/\Sigma_U)^{1/2}$ and $\beta = (\Sigma_U/\Sigma_V)^{1/2}$, and Σ_V and Σ_U are the unconditional variances of the payoff and uninformed-trader demand, respectively.

The other direction of this question (assume the informed trader strategy and show marketmaker strategy is optimal) is presented in the textbook. The direction in this question is actually simpler. The informed trader wants to choose D^I to maximize his expected profit, or $\text{Max } E[D^I(V - P)]$. Substituting the marketmaker's pricing strategy into this expression for expected profit we get $\text{Max } E[D^I(V - \lambda(D^I + D^U))]$. With the expected value of D^U being zero, the first order condition reduces to: $D^I = V/(2\lambda)$. The term $1/(2\lambda)$ is precisely equal to the expression for β in the problem, so we're done.

True, False, Uncertain: Explain

- Market transparency refers to the degree to which price is observable by market participants.

False, in the sense that transparency is much broader than this. (See the discussion in the text, pages 55-57.) At the very least, it involves also quantity (e.g., order flow) information and the distinction whether price and quantity information are available both pre and post trade.

- Order flow analysis and fundamental analysis are different.

Uncertain. The answer turns on how one defines "fundamentals". As we discussed in class, even if one takes a narrow definition of FX fundamentals to include only those drivers of exchange rates that would enter in, say, a flexible-price monetary model (which assumes the risk premium is zero), order flow might still convey information about future money supplies, interest rates, etc. For example, order flow could convey information to dealers about people's expectations of future fundamentals, which are not common knowledge as they are evolving. (Think of an environment in which some agents are better than others at processing current macro information and forming more precise expectations about future macro policy.)

If one is willing to include the effects of risk on exchange rates (and the variables that drive risk premia) as "fundamental," then it is easy to establish close links between order flow and fundamentals. In this case, order flow just needs to convey any information that would be relevant in, say, a portfolio balance model, so long as that information is not yet known by all. Examples include shocks to individuals' hedging demands or shocks to firms' risk tolerances.

- In the Glosten-Milgrom model (JFE 1985), the marketmaker's current bid and ask quotes reflect the information in the previous incoming trade, but not the information in the next incoming trade.

False: they include the information in the next incoming trade as well. Conceptually, this is a very important feature of the model. It is what establishes the marketmaker's quotes as "regret free" (i.e., consistent with rational expectations). See the discussion on pages 88-89 of the textbook.